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ONSET OF COHERENT LARGE-SCALE MOTION IN A PLANE TURBULENT WAKE

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In the present article we give the results of a theoretical investigation of the response of a plane (two-dimensional) turbulent wake to an external harmonic disturbance. The underlying concepts and the approach used for the stated problem are discussed in [1]. Apart from the fact that the flow geometry differs from [1], we also consider the influence of crossflow variation of the turbulent viscosity on the evolution of large-scale disturbances.

1. Self-Similar Plane Turbulent Wake. Following [1], we introduce the turbulent Reynolds number for a self-similar wake:

$$\mathrm{Re}_{\mathrm{T}} = u_0 b/\nu_{\mathrm{T}} (\equiv \mathrm{const}), \tag{1.1}$$

where $\nu_T(X) \sim u_0 b$ is a characteristic turbulent viscosity in the cross section of the wake at the longitudinal coordinate $X = (x - x_0)$ measured from a fictitious origin x_0, and u_0 and b are local velocity and length scales. The length scale is given by the relation

$$b = (\nu_{\mathrm{T}} X/U_\infty)^{1/2}. \tag{1.2}$$

The resistance offered by the body against a flow with velocity U_∞ has the form

$$F = \rho \int_{-\infty}^{\infty} U(U - U_\infty)\,dy \quad (\equiv \rho U_\infty^2 \theta) \tag{1.3}$$

(θ is the momentum loss thickness). We represent the average flow velocity in self-similar far-wakes by the expression

$$U = U_\infty[1 - \varepsilon\varphi_0(\eta)],\ V = U_\infty \varepsilon^2 \chi_0(\eta) \tag{1.4}$$

($\varepsilon = u_0/U_\infty \ll 1$ and $\eta = y/b$ is the dimensionless transverse coordinate). We rewrite Eq. (1.3):

$$\theta = \varepsilon b J_1 - \varepsilon^2 b J_2,\ J_n = \int_{-\infty}^{\infty} \varphi_0^n(\eta)\,d\eta, \quad n = 1,\ 2. \tag{1.5}$$

Disregarding the term $O(\varepsilon^2)$ in Eq. (1.5) and making use of Eqs. (1.1) and (1.2), we obtain expressions for the local scales:

$$u_0/U_\infty = C(X/\mathrm{Re}_{\mathrm{T}})^{-1/2},\ b = C(X/\mathrm{Re}_{\mathrm{T}})^{1/2},\ C = (\theta/J_1)^{1/2}. \tag{1.6}$$

We express Re_T in a form suitable for experimental evaluation:

$$\mathrm{Re}_{\mathrm{T}} = (X u_0)/(b U_\infty). \tag{1.7}$$

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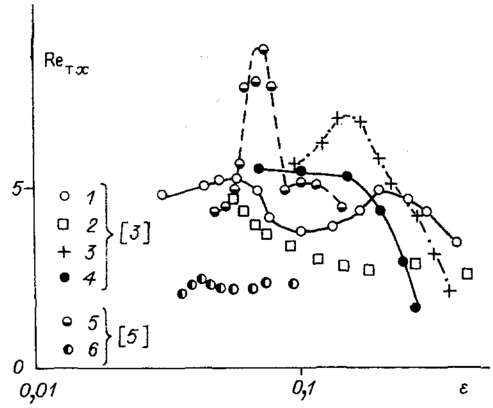

Fig. 1

According to the flow self-similarity condition, the Reynolds stresses have the form

$$\overline{u'v'} = u_0^2\sigma(\eta). \tag{1.8}$$

Disregarding viscous stresses and terms $O(\varepsilon^2)$ and making use of Eqs. (1.7) and (1.8), we obtain the following relation from the average equations of motion:

$$(1/2)(\varphi_0 + \eta\varphi_0') = \mathrm{Re}_T\,\sigma' \tag{1.9}$$

(the prime denotes differentiation with respect to η). Integrating Eq. (1.9), we obtain an expression for the self-similar Reynolds stresses:

$$\sigma = (2\mathrm{Re}_T)^{-1}\eta\varphi_0. \tag{1.10}$$

The system is closed by the following relation, which expresses the Boussinesq hypothesis of turbulent viscosity:

$$\sigma = (\mu_T \partial U/\partial y)/(\rho u_0^2). \tag{1.11}$$

From relations (1.10) and (1.11) we obtain an equation for the self-similar average-velocity deficit:

$$\varphi_0 = \exp(-\ln 2\cdot\eta^2), \tag{1.12}$$

where the transverse velocity scale is chosen in accordance with [1]. We denote it by L_0. The transverse component of the velocity is determined from the equation of continuity and Eqs. (1.4) and (1.7): $\chi_0 = \eta\varphi_0/2\mathrm{Re}_T$. The experimentally determined average velocity differs only very slightly from the functional relation deduced from Eqs. (1.4) and (1.12); it has the form [2]

$$U/U_\infty = 1 - \varepsilon\exp(-A\eta^2 - B\eta^4),\ A = 0{,}637,\ B = 0{,}056. \tag{1.13}$$

An expression for the turbulent viscosity is obtained from relations (1.10) and (1.11):

$$\mu_T/(\rho\nu_T) = A/(A + 2B\eta^2)\ (\equiv N). \tag{1.14}$$

The role of viscosity effects can be estimated from Eqs. (1.1) and (1.5): $\nu/\nu_T = J_1(\mathrm{Re}_T/\mathrm{Re}_\theta)$ ($\mathrm{Re}_\theta = U_\infty/\nu$). The constant J_1, which is calculated according to Eqs. (1.12) and (1.13), has the values $(\pi/\ln 2)^{1/2} = 2.13$ and 2.06. Experimental studies [2, 3] give $J_1 = 2.06$.

We use experimental data to determine the values of Re_T. Sreenivasan [3] and Narasimha and Prabhu [4] have introduced the parameters

$$W = (u_0/U_\infty)(x/\theta)^{1/2},\ \Delta = L_0(x\theta)^{-1/2}, \tag{1.15}$$

whose variation in the flow direction should exhibit the tendency of the motion in a plane turbulent wake toward universally self-similar flow. By analogy with Eq. (1.7), we introduce the parameter $\mathrm{Re}_{Tx} = W/\Delta$, which corresponds to the replacement of X by x. This relation gives the local value of Re_T, as opposed to Eq. (1.7), which gives the average value of Re_T characterizing the intermediate self-similar regime. Figure 1 shows Re_{Tx} as a function of the relative average-velocity deficit for various plane turbulent wake patterns (points 1 and

2 correspond to circular and square cylinders, points 3 and 4 to single- and two-layer flat plates, points 5 to a circular cylinder, and points 6 to a circular cylinder in a turbulent flow); Re_{TX} varies as the wake evolves, and for some generators its variation is nonmonotonic. The local values can also be determined from measurements of the Reynolds stresses by means of Eq. (1.10) (see Fig. 1 and the results of [5]). The evolution of the flow behind bodies of various configurations retains its individuality, evidently because of the existence of large-scale disturbances, the nature of whose development depends on Re_{TX} and on the presence of external disturbances. When the background turbulence contains disturbances approaching the most dangerous kind for the given flow pattern in the wake, turbulent transfer can be intensified considerably because of the development and subsequent disintegration of large-scale disturbances [5]. Experimental values for a large number of plane turbulent flow patterns are given in [2], including (in particular) the average values of the parameters W_0 and Δ_0 [which are analogous to (1.15), but with X in place of x]; the values of $\mathrm{Re}_T = W_0/\Delta_0$ calculated from them lie in the interval 3.46-6.96. The boundary values of Re_T are those for the wake of a flat plate with superimposed external harmonic disturbances and a solid flat baffle. Equation (1.15) can be used in place of (1.5) to obtain a linear approximation relating the scales u_0 and L_0: $\theta = \varepsilon L_0/\Delta_0 W_0$. The average value $1/\Delta_0 W_0 = 1.97$ can be substituted for the integral J_1 in the expressions for the local scales (1.6), which describe the existing experimental data very well in this case.

2. Statement of the Problem. A method for investigating the linear response of turbulent wakes to an external harmonic disturbance is described in detail in [1]. We merely note certain departures associated with the freestream geometry and the variable turbulent viscosity. As in [1], we seek solutions of the linear response equations in the form

$$\left\{\begin{matrix}(\tilde{u}, \tilde{v}, \tilde{w}) \\ \tilde{p}\end{matrix}\right\} = \left\{\begin{matrix}\varepsilon U_\infty [u(\eta), v(\eta), w(\eta)] \\ \varepsilon^2 U_\infty^2 q(\eta)/\mathrm{Re}_T\end{matrix}\right\} \exp(i\Theta),$$

where notation has been introduced for the velocity components (u, v, w) corresponding to the Cartesian coordinates (x, y, z). We specify the phase of the disturbances by the relations

$$\partial\Theta/\partial X = \alpha^0 + (\varepsilon/\mathrm{Re}_T)\alpha_1^0(X), \quad \partial\Theta/\partial z = \beta^0, \quad \partial\Theta/\partial t = -\omega^0 \tag{2.1}$$

(α^0 and β^0 are the longitudinal and transverse wave numbers). According to Eq. (2.1), the phase of the disturbances has the form

$$\Theta = (\alpha^0 X + \beta^0 z - \omega^0 t) + \left(\int \varepsilon\alpha_1^0(X)\, dX\right)\Big/ \mathrm{Re}_T.$$

Relying on assumptions set forth in [1], we obtain a system of ordinary differential equations in the amplitude functions of the perturbed motion. The Squire transformation reduces this system to a single fourth-order equation in the transverse component of the velocity of the disturbance:

$$\begin{gathered} i[(\alpha_1 - \alpha\mathrm{Re}_T\varphi_0)(D^2 - k^2) + \alpha\mathrm{Re}_T D^2\varphi_0]v - (1/2)[\eta(D^3 - k^2 D) + \\ + 2D^2 - k^2]v = N(D^2 - k^2)^2 v + 2(DN)(D^3 - k^2 D)v + (D^2 N)(D^2 + k^2)v \end{gathered} \tag{2.2}$$

($D \equiv d/d\eta$ and $k^2 = \alpha^2 + \beta^2$). The extinction condition on the disturbances at infinity can be augmented by symmetry conditions on the axis, which permits the solution of Eq. (2.2) to be analyzed on the interval $[0, \infty]$. We write the corresponding boundary conditions as

$$\begin{gathered} v, Dv \to 0 \text{ as } \eta \to \infty, \\ v(0) = D^2 v(0) = 0 \text{ for symmetrical disturbances;} \\ Dv(0) = D^3 v(0) = 0 \text{ for antisymmetrical disturbances.} \end{gathered} \tag{2.3}$$

The problem of the nature of the evolution of small harmonic disturbances in a turbulent wake is solved by finding the eigenvalues α_1 and eigenfunctions of the boundary-value problem (2.2), (2.3). The method of solution and certain essentials associated with the boundary conditions at infinity are discussed in [1]. The numerical algorithm was tested using data from [6]. Problem (2.2), (2.3) is reduced to the well-studied case of a plane laminar wake in the parallel-flow approximation by substituting $-U$ for φ_0 and introducing the phase velocity $c = -\alpha_1/\alpha Re_T$. Furthermore, it is required to set $N = 1$ and to eliminate the second term on the left-hand side of Eq. (2.2), which is the term associated with nonparallel flow in the wake.

The amplitude of the disturbances varies along the flow according to a power law [1]:

$$(\tilde{u}, \tilde{v}, \tilde{w})/U_\infty \sim \varepsilon X^{i\alpha_1}. \qquad (2.4)$$

In addition to the self-similar part $\varepsilon \sim X^{-1/2}$, the exponent also contains a number characterizing the decay or growth of the disturbances. This number depends on the wavelength of the disturbances and on Re_T. It follows from Eq. (2.4) that the intensity of the disturbances decreases downstream for $-0.5 < \alpha_{1i} < 0$, but their amplitude increases relative to the average motion. This behavior causes the disturbances to exert a major influence on the average flow.

3. Results of Numerical Calculations and Discussion. In addition to the average velocity profile, Eq. (2.2) also contains a variable viscosity, which determines the influence of turbulence on the evolution of large-scale disturbances. In the case of free shear flows the role of viscosity is essentially that of stabilizing the disturbances, so that allowance for the variation of the effective crossflow viscosity does little more than abate this process, since the turbulent viscosity decreases toward the periphery of the flow. The foregoing discussion sheds light on the modification of the results when variable viscosity is taken into account. Because of certain computational difficulties associated with the boundary conditions at the outer boundary of the flow, we first investigate a constant turbulent ($N \equiv 1$) in detail. According to Squire's theorem, the nature of the evolution of three-dimensional disturbances can be estimated by analyzing two-dimensional disturbances. We therefore consider the case $\beta = 0$. We know from the theory of hydrodynamic stability that two types of unstable disturbances exist for plane free shear flows: symmetrical and antisymmetrical. As a rule, symmetrical disturbances do not pose as serious a threat.

Calculations in the present study have shown that turbulent wake flow is unstable against disturbances of this type for $Re_T > 37.7$. The variable viscosity model (1.14) gives a much smaller critical Reynolds number $Re_{T*} = 22.8$ for $\alpha_* = 0.352$ and c $(\equiv -\alpha_{1r}/\alpha Re_T) = -0.76$ (the phase velocity of these disturbances is equal to 3/4 of the maximum average-velocity deficit). Even in the given situation, however, the experimental values of Re_T are still smaller than this value. We can therefore conclude that a plane turbulent wake is stable against small symmetrical large-scale disturbances. Structures of both symmetries have been observed in experimental investigations of coherent structures in a plane turbulent wake [7, 8], even though the antisymmetrical type is more probable. The presence of symmetrical coherent structures is evidently a consequence of their nonlinear, possibly resonance interaction with antisymmetrical structures.

Figure 2 shows the domains of existence of stable and unstable antisymmetrical disturbances. The smallest Reynolds number at which neutral disturbances exist is equal to 2.66, which corresponds to $\alpha_* = 0.17$ and $\alpha_{1r} = 0.1$ $(c = -0.22)$. Figure 3 shows the eigenvalue α_1 as a function of the wave number α for $Re_T = 1$ (curve 1) and $Re_T = 7$ (curve 2). In the limit $Re_T \to 0$ the eiganvalue acquires the asymptotic form $\alpha_1 = i\alpha^2$. The neutral curve for the model viscosity (1.14) is represented by the dot-dashed curve in Fig. 2. The critical value of Re_T cannot be calculated, because as α decreases, the outer boundary must be superseded by no-slip conditions on it, and this incurs a loss of computational accuracy. The changes in behavior of this neutral curve is fully consistent with the foregoing discussion.

It has been observed and recorded in turbulent flow around cylinders of various geometries [5] that the wake of the generator evolved self-similarly in accordance with relation (1.6). However, its width increased far more rapidly when large-scale disturbances were present in the freestream flow. On the other hand, the presence of small-scale disturbances commensurate in size with the diameter of the cylinder did not influence the evolution of the wake. External large-scale disturbances intensify turbulent transfer in the wake, stimulating the onset of internal hydrodynamic instabilities. In the case of an undisturbed freestream flow the role of the external influence is fulfilled by wake-generation conditions such as the presence of disturbances in the boundary layer on the body and flow separation accompanied by the formation of vortex structures.

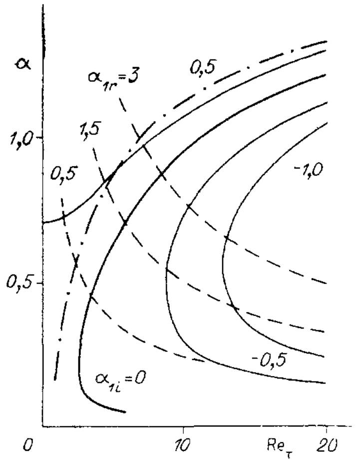

Fig. 2

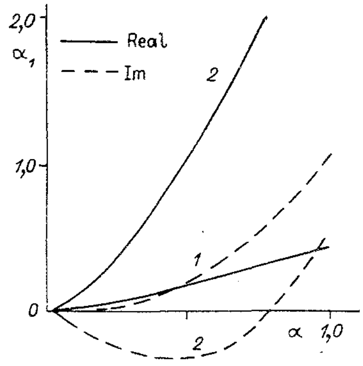

Fig. 3

The instability of turbulent free shear flows against large-scale disturbances is evidently the mechanism responsible for maintaining the necessary condition for turbulent fluctuations and for the downstream growth of their scale. The linear theory of hydrodynamic stability of viscous flows can be used to determine the characteristic scale of the most dangerous external disturbances, which stimulate the rapid evolution of flow toward the limiting self-similar regime.

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